

Maximizing Acceptance Probability for Active Friending in On-Line Social Networks

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ABSTRACT

Friending recommendation has successfully contributed to the explosive growth of on-line social networks. Most friending recommendation services today aim to support passive friending, where a user passively selects friending targets from the recommended candidates. In this paper, we advocate recommendation support for active friending, where a user actively specifies a friending target. To the best of our knowledge, a recommendation designed to provide guidance for a user to systematically approach his friending target, has not been explored in existing on-line social networking services. To maximize the probability that the friending target would accept an invitation from the user, we formulate a new optimization problem, namely, *Acceptance Probability Maximization (APM)*, and develop a polynomial time algorithm, called *Selective Invitation with Tree and In-Node Aggregation (SITINA)*, to find the optimal solution. We implement an active friending service with SITINA in Facebook to validate our idea. Our user study and experimental results manifest that SITINA outperforms manual selection and the baseline approach in solution quality efficiently.

Keywords

Friending, social network, social influence

1. INTRODUCTION

Due to the development and popularity of social networking services, such as Facebook, Google+, and LinkedIn, the new notion of “social network friending” has appeared in recent years. To boost the growth of their user bases, existing social networking services usually provide friending recommendations to their users, encouraging them to send invitations to make more friends. Conventionally, friending recommendations are made following a *passive friending* strategy, i.e., a user passively selects candidates from the provided recommendation list to send the invitations. Moreover, the recommended candidates are usually friends-of-friends of the user, especially those who share many com-

mon friends with the user. This strategy is quite intuitive because friends-of-friends may have been acquaintances or friends offline. Furthermore, most users may feel more comfortable to send a friending invitation to friends-of-friends rather than a total stranger who they have shared no social connections with at all. It is envisaged that the success rate of such a passive friending strategy is high, contributing to the explosive growth of on-line social networking services.

In contrast to the passive friending, the idea of *active friending*, where a person may take proactive actions to make friend with another person, does exist in our everyday life. For example, in a high school, a student fan may like to make friend with the captain in the school soccer team or with the lead singer in a rock-and-roll band of the school. A salesperson may be interested in getting acquainted with a high-value potential customer in hope of making a business pitch. A young KDD researcher may desire to make friend with the leaders of the community to participate in conference organizations and services. However, to the best of our knowledge, the idea of providing friending recommendations to assist and guide a user to effectively approaching another person for active friending has not been explored in existing on-line social networking services. We argue that social networking service providers, interested in exploring new revenues and further growth of their user bases, may be interested in supporting active friending.

One may argue that, in existing social networking services, an *active friending initiator* can send an invitation directly to the *friending target* anyway.¹ However, it may not work if the initiator is regarded as a stranger by the target, especially when they are socially distant, i.e., they have no common friends. Therefore, to increase the chance that the target would accept the friending invitation, it may be a good idea for the initiator to first know some friends of the target, which in turn may require the initiator to know some friends of friends of the target. In other words, if the initiator would like to plan for some actions, he may need the topological information of the social network between the target and himself, which unfortunately is not available due to privacy concerns. Therefore, it would be very nice if the social networking service providers, given a target specified by the initiator, could provide a step-by-step guidance in form of recommendations to assist the initiator to make friends towards the target.

In this paper, we are making a grand suggestion for the social networking service providers to support active friend-

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¹For the rest of the paper, we refer to the friending initiator and friending target as *initiator* and *target* for short.

ing. Our sketch is as follows. By iteratively recommending a list of candidates who are friends of at least one existing friend of the initiator, a social networking service provider may support active friending, without violating the current practice of privacy preservation in recommendations. Consider an initiator who specifies a friending target. The social networking service, based on its proprietary algorithms, recommends a set of friending candidates who may likely increase the chance for the target to accept the eventual invitation from the initiator. Similar to the recommendations for passive friending, the recommendation list consists of only the friends of existing friends of the initiator. Supposedly, the initiator follows the recommendations to send invitations to candidates in the list. The invitation is displayed to a candidate along with the list of common friends between the initiator and the candidate so as to encourage acceptance of the invitation.² As such, the aforementioned step is repeated until the friending target appears in the recommendation list and an invitation is sent by the initiator. Obviously, the recommendations made for passive friending may not work well because active friending is target-oriented. The recommended candidates should be carefully chosen for the initiator, guiding him to approach the friending target step-by-step.

To support active friending, the key issue is on the design of the algorithms that select the recommendation candidates. A simple scheme is to provide recommendations by unveiling the shortest path between the initiator and the target in the social network, i.e., recommending one candidate at each step along the path. As such, the initiator can gradually approach the target by acquainting the individuals on the path. However, this shortest-path recommendation approach may fail as soon as a middle-person does not accept the friending invitation (since only one candidate is included in the recommendation list for each step). To address this issue, it is desirable to recommend multiple candidates at each step since the initiator is more likely to share more common friends with the target and thereby more likely to get accepted by the target. Especially, by *broadcasting* the friending invitations to all neighbors of the initiator's friends, the probability to reach the friending target and get accepted can be effectively maximized as enormous number of paths are flooded with invitations to approach the target. Nevertheless, friending invitations are abused here because the above undirectional broadcast is aimless and prone to involve many unnecessary neighbors. Moreover, the initiator may not want to handle a large number of tedious invitations.

In this paper, we study a new optimization problem, called *Acceptance Probability Maximization (APM)*, for active friending in on-line social networks. The service providers, who eager to explore new monetary tools for revenue increase, may consider to charge the users from active friending service.³ Given an initiator s , a friending target t , and the maximal number r_R of invitations allowed to be issued by the initiator, APM finds a set R of r_R nodes, such that s can sequentially send invitations to the nodes in R in order to approach t . The objective is to maximize the acceptance

probability at t of the friending invitation when s send it to t . The parameter r_R controls the trade-off between the expected acceptance probability of t and the anticipated efforts made by s for active friending t .⁴ Again, R is not returned to s as a whole due to privacy concerns. Instead, only a subset R_s of nodes that are adjacent to the existing friends of s are recommended to s , while other subsets of R will be recommended to s as appropriate in later steps⁵.

To tackle the APM problem, we propose three algorithms: i) Range-based Greedy (RG) algorithm, ii) Selective Invitation with Tree Aggregation (SITA) algorithm, and iii) Selective Invitation with Tree and In-Node Aggregation (SITINA) algorithm. RG selects candidates by taking into account their acceptance probability and the remaining budget of invitations, leading to the best recommendations for each step. However, the algorithm does not achieve the optimal acceptance probability of the invitation to a target due to the lack of coordinated friending efforts. On the other hand, aiming to systematically select the nodes for recommendation, SITA is designed by dynamic programming to find nodes which may result in a coordinated friending effort to increase the acceptance probability of the target. SITA is able to obtain the optimal solution, yet has an exponential time complexity. To address the efficiency issue, SITINA further refines the ideas in SITA by carefully aggregating some information gathered during processing to alleviate redundant computation in future steps and thus obtains the optimal solution for APM in polynomial time. The contributions of this paper are summarized as follows.

- We advocate for the idea of *active friending* in on-line social networks and propose to support active friending through a series of recommendation lists which serve as a step-to-step guidance for the initiator.
- We formulate a new optimization problem, namely, Acceptance Probability Maximization (APM), for configuring the recommendation lists in the active friending process. APM aims to maximize the acceptance probability of the invitation from the initiator to the friending target, by recommending selective intermediate friends to approach the target.
- We propose a number of new algorithms for APM. Among them, Selective Invitation with Tree and In-Node Aggregation (SITINA) derives the optimal solution for APM with $O(n_V r_R^2)$ time, where n_V is the number of nodes in a social network, and r_R is the number of invitations budgeted for APM.
- We implement SITINA in Facebook in support of active friending and conduct a user study including 169

⁴Since s is not aware of the network topology and the distance to t , it is not reasonable to let s directly specify r_R . Instead, it is more promising for the service provider to list a set of r_R and the corresponding acceptance probabilities and monetary costs, so that the user can choose a proper r_R according to her available budget.

⁵In this paper, APM is formulated as an offline optimization problem aiming to maximize the acceptance probability in expectation. In an on-line scenario where the initiator does not send invitations to some nodes in R_s or some nodes in R_s do not accept the invitations, a new APM with renewed invitation budget could be re-issued to obtain adapted recommendations. While this scenario raises important issues, it is beyond the scope of this paper.

²This is also a common practice for passive friending in existing social networking services such as Facebook, Google+, and LinkedIn.

³Recent news reported that Facebook now allows its user to pay to promote their and their friends' posts [1].

volunteers with varied background. The user study and experimental results manifest that SITINA outperforms manual selection and the baseline approach in solution quality efficiently.

The rest of this paper is organized as follows. Section 2 introduces a model for invitation acceptance and formulates APM. Section 3 reviews the related work. Section 4 presents the SITA and SITINA algorithms proposed for APM. Section 5 reports our user study and experimental results. Finally, Section 6 concludes the paper.

2. INVITATION ACCEPTANCE

The notion of *acceptance probability* is with respect to an invitation. Thus, here we first discuss two important factors that may affect the acceptance probability of a friending invitation in the environment of on-line social networking services and describe how in this work we determine whether an individual would accept a received invitation. Next, we explain why the issue of deriving the acceptance probability over a social network is very challenging and how we address this issue by adopting an approximate probability based on a maximum influence in-arborescence (MIIA) tree. We formulate the acceptance probability maximization (APM) problem based on the MIIA tree. The invitation acceptance model follows the existing social influence and homophily models, which have been justified in the literature. Later in Section 5, the invitation acceptance model will be validated by a user study with 169 volunteers.

2.1 Factors for Invitation Acceptance

In the process of active friending, while friending candidates are recommended for the initiator to send invitations, whether the invitees will accept the invitations remains uncertain. Based on prior research in sociology and on-line social networks [12, 20, 21], we argue that when a person receives an invitation over an on-line social network, the decision of the invitee depends primarily on two important factors: i) the *social influence factor* [13, 14], and ii) the *homophily factor* [9, 12, 21]. Here, the social influence factor represents the influence from the surroundings (i.e., common friends) of individuals in the social network on the decision. On the other hand, the homophily factor captures the fact that each individual in a social network has a distinctive set of personal characteristics, and the similarities and compatibilities among characteristics of two individuals can strongly influence whether they will become friends [12]. Between them, social influence comes from established social links, while the homophily between two individuals may exist without a prerequisite of established social relationship. Thus, we consider these two factors separately but aim to treat them in a uniformed fashion in our derivation of the acceptance probability for an invitation.

As the social influence factor involves the structure of social network (i.e., the common friends of the individuals), we first consider the acceptance probability of an invitation in terms of social influence.⁶ Let the social network be represented as a social graph $G(V, E)$ where V consists of all the users in the social networking system and E be the established social links among the users. An edge weight $w_{u,v} \in [0, 1]$ on the directed edge $(u, v) \in E$ probabilistically

⁶We intend to extend it with homophily factor later.

denotes the social influence of u upon v . The probability can be derived according to the existing method [13, 14] according to the interaction in on-line social networks, while the setting of negative social influence has also been introduced in [4]. Thus, if u is associated with an invitation from a user s to v (i.e., u is a common friend of s and v), $w_{u,v}$ is the probability for v to be socially influenced by u to accept the invitation.⁷ Hence, the acceptance probability for an invitation can be derived by taking into account the social influences of all the existing common friends associated with an invitation. It is assumed that each common friend u has an independent social influence on the invitee v to accept the friending invitation [9, 12, 20] and thus the overall acceptance probability can be obtained by aggregating the individual social influences. Later, user study in Section 5 demonstrates that the influence probability and homophily probability derived according to the literature are consistent to the real probabilities measured from the users.

While obtaining the acceptance probability for a given invitation (as described above) is simple, deriving the acceptance probability for a friending target t who does not have any common friend with the initiator s becomes very challenging because more than one invitations need to be issued (so as to make some common friends first), and there are complicated correlations among user acceptance events for users between s and t .

Moreover, our ultimate task is to find a set R of intermediate users between s and t with size at most r_R for s to send invitations to, so as to maximize the acceptance probability of t . We call this problem the acceptance probability maximization (APM) problem. Due to the combinatorial nature of this invitation set R , it is still hard to find such a set to maximize the acceptance probability of t even in cases where computing the acceptance probability is easy. The following theorem makes the above two hardness precise.

THEOREM 1. *Given the set of neighbors S of the initiator, computing the acceptance probability of t is $\#P$ -hard. Moreover, finding a set R with size r_R that maximizes the acceptance probability of target t is NP -hard, even for cases when computing acceptance probability is easy.*

PROOF. We first prove that computing the acceptance probability of t with given R is $\#P$ -hard. Let G_R denote the induced subgraph of G with s , t , and R . Let \overline{G}_R denote a directed subgraph of G_R by removing every edge (u, v) in G_R if u does not influence v to accept an invitation, either because u does not become a friend of s or v does not accept the invitation due to the social influence from u . Therefore, t will finally be a friend of s if there exists a path in \overline{G}_R , representing that every node in the path, including t , accepts the friend invitation from s . Apparently, if the probability of social influence associated with each edge is 0.5, the probability that t accepts the friend invitation is the number of subgraph \overline{G}_R with t accepting, divided by the number of possible subgraph \overline{G}_R , which is 2^{n_E} , where n_E is the number of directed edges. In other words, after acquiring the accepting probability, the number of subgraph \overline{G}_R with t accepting can be computed immediately by multiplying 2^{n_E} .

⁷The social influence probability has been extensively used to quantify the probability of success in the process of conformity, assimilation, and persuasion in Social Psychology [9, 12, 20]. While how to obtain the edge weight is an active research topic [13, 24], it is out of scope of this paper.

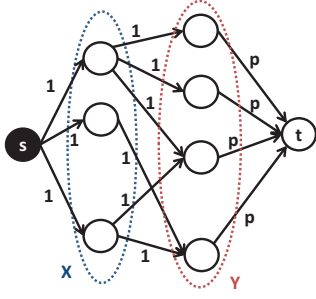


Figure 1: An illustration graph of building the APM instance

We prove that computing the acceptance probability of t is $\#P$ -hard with the reduction from a $\#P$ -complete problem, called *s-t connectedness problem* [27], that finds the number of subgraphs in a directed graph G_C with a directed path from s to t . We let $G_R = G_C$ and assign the probability of social influence with each edge as 0.5. With the observation in the previous paragraph, if finding the accepting probability of t is not $\#P$ -complete, *s-t connectedness problem* is not $\#P$ -complete because the number of subgraphs in G_C with a directed path from s to t is simply the accepting probability of t multiplied by 2^{n_E} .

Moreover, even for cases when computing the acceptance probability of t is easy, finding a set R that maximize the acceptance probability of target t is NP-hard in IC model. We prove it with a reduction from the *set cover problem*. For a bipartite graph (X, Y, E) , set cover problem aims to identify whether there exists a k -node subset X_S of X covering all nodes in Y , i.e., for any $y \in Y$, there exists an $x \in X_S$ with $(x, y) \in E$. Let us denote $|Y| = z_y$. For an instance of set cover problem, we build an instance for computing the acceptance probability of t as follows, and an illustration figure is shown in Figure 1. 1) We add a node s and a directed edge (s, x) for each $x \in X$ with weight $w(s, x) = 1$. Notice that s is the only one node with acceptance probability 1 in the beginning. 2) We add a node t and a directed edge (y, t) for each $y \in Y$ with weight $w(y, t) = p$, $p \in (0, 1)$. 3) We set the $w(e) = 1$ for each $e \in E$. We prove that there is a k -node subset $X_S \subseteq X$ covering all nodes in Y in the set cover problem if and only if there is a solution with acceptance probability $1 - (1 - p)^{z_y}$ when selecting $r_R = k + z_y + 1$ nodes in computing the acceptance probability.⁸ We first prove the sufficient condition. If there exists a k -node subset X_S covering all nodes in Y , selecting $X_S \cup Y \cup \{t\}$ (totally $k + z_y + 1$ nodes) will obtain acceptance probability $1 - (1 - p)^{z_y}$. We then prove the necessary condition. If there is a solution R with r_R nodes obtaining acceptance probability $1 - (1 - p)^{z_y}$, R must contain t and all nodes in Y , and the set $R \cap X$ (totally $r_R - z_y - 1 = k$ nodes) must cover all nodes in Y . Thus selecting a suitable R is NP-hard. The theorem follows. \square

2.2 Approximate Acceptance Probability

The spread maximization problem in Independent Cascade (IC) model [17] also faces the challenge in Theorem 1.⁹

⁸Notice that $1 - (1 - p)^{z_y}$ is the maximum probability when including all nodes in $X \cup Y \cup t$ into R , thus it is obvious the maximum probability when selecting r_R nodes.

⁹The spread maximization problem, which also adopts a

To efficiently address this issue, an approximate IC model, called MIA [4, 5, 6], has been proposed. The social influence from a person u to another person v is effectively approximated by their *maximum influence path* (MIP), where the social influence $w_{u,v}$ on the path (u, v) is the maximum weight among all the possible paths from u to v . MIA creates a maximum influence in-arborescence, i.e., a directed tree, $MIIA(t, \theta)$ including the union of every MIP to t with the probability of social influence at least θ from a set S of leaf nodes. The MIA model has been widely adopted to describe the social influence in the literature [4, 5, 6] with the following definition on activation probability, which basically is the same as the acceptance probability if s broadcasts friending invitations to all nodes in $MIIA(t, \theta)$.

DEFINITION 1. The activation probability of a node v in $MIIA(t, \theta)$ is $ap'(v, S, MIIA(t, \theta)) =$

$$\begin{cases} 1, & \text{if } v \in S \\ 0, & \text{if } N^{in}(v) = \emptyset \\ 1 - \prod_{u \in N^{in}(v)} (1 - ap'(u, S, MIIA(t, \theta)) \cdot w_{u,v}), & \text{otherwise} \end{cases}$$

where $N^{in}(v)$ is the set of in-neighbors of v .

Note that $ap'(u, S, MIIA(t, \theta)) \cdot w_{u,v}$ is the joint probability that u is activated and successfully influences v , and u can never influence v if it is not activated. Therefore, the activation probability of a node v can be derived according to the activation probability of all its in-neighbors, i.e., child nodes in the tree. Since S is the set of leaf nodes, the activation probabilities of all nodes in $MIIA(t, \theta)$ can be derived in a bottom-up manner from S toward t efficiently.

In light of the similarity between the IC model and the decision model for invitation acceptance in active friending with no budget limitation of invitations, we also exploit MIA to tackle the APM problem. $MIIA(t, \theta)$ is constructed by the MIPs from all friends of s to t , i.e., S is the set of friends of s . In other words, θ is set as 0 to ensure that the social influence from every friend is fully incorporated. Nevertheless, different from the activation probability in the literature, which allows the influence to propagate via every node in $MIIA(t, \theta)$, the *acceptance probability* for active friending allows the social influence to take effect on invitation acceptance only via a set R of nodes to be selected in our problem. Thus, we define the acceptance probability for an invitation to node v as follows.

DEFINITION 2. The acceptance probability for an invitation of a node v in $MIIA(t, \theta)$ is $ap(v, S, R, MIIA(t, \theta)) =$

$$\begin{cases} 1, & \text{if } v \in S \\ 0, & \text{if } v \notin R \text{ or } N^{in}(v) = \emptyset \\ 1 - \prod_{u \in N^{in}(v), u \in R} (1 - ap(u, S, R, MIIA(t, \theta)) \cdot w_{u,v}), & \text{otherwise} \end{cases}$$

where $N^{in}(s)$ is the set of in-neighbors of s .

Equipped with MIA, we are able to derive the acceptance probability of t efficiently with a simple iterative approach from the leaf nodes to the root (i.e., t). The above MIA arborescence incorporates only the social influence factor. As probabilistic influence model, is different from APM in this paper. Given an initiator s and his friends, APM intends to discover an effective subgraph (i.e., R) between the seeds and t . On the other hand, the spread maximization problem, given the topology of the whole social network, aims to find a given number of seeds to maximize the size of the whole spread t .

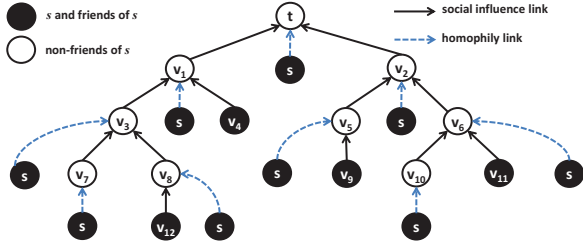


Figure 2: Combining the social influence and homophily factors

discussed earlier, the homophily factor between the initiator and the receiver of an invitation is also crucial for friending. Homophily in [9, 12, 21] represents the probability for two individuals u and v to create a new social link due to shared common personal characteristics. Homophily in Sociology manifests the general tendency of people to associate with others and similar others can be quantified with varied approaches [2, 16, 30]. The homophily probability can be set according to [3].

To extend MIA, we attach a duplicated s to each node with a directed edge, with a parameter specifying the homophily factor from s to v . The MIP from each candidate to t , together with the directed edge from s to the candidate, is incorporated in the extended MIA. Therefore, the extended MIA is also an arborescence, where each leaf node is a friend of s or s herself, and those leaf nodes make up the set S .

Figure 2 shows an example of the extended MIA. For each internal node, such as v_1 , its acceptance probability factors is not only the social influence from v_3 and v_4 but also the homophily factor between s and v_1 .

In this paper, the influence probability and homophily probability are derived according to the above literature without associating them with different weights. Later, user study will be presented in Section 5, and the results show that the real acceptance probability complies with the acceptance probability of the above model.

2.3 Problem Formulation

In this work, we formulate an optimization problem, called *Acceptance Probability Maximization (APM)*, to select a given number of intermediate people to systematically approach the friending target t based on $MIIA(t, \theta)$. The APM problem is formally defined as follows.

Acceptance Probability Maximization (APM). Given a social network $G(V, E)$, an initiator s and a friending target t , select a set R of r_R users for s to send friending invitations such that the acceptance probability $ap(t, S, R, MIIA(v, \theta))$ is maximized, where S is the friends of s , including s itself.

As analyzed later, the optimal solution to APM can be obtained in $O(n_V r_R^2)$ time¹⁰, where n_V is the number of nodes in a social network, and r_R is the total number of invitations allowed. The setting of r_R has been discussed in Section 1. It is worth noting that APM maximizes the acceptance probability of t , instead of minimizing the number

of iterations to approach t , which can be achieved by the shortest path routing in an on-line social network. Nevertheless, it is possible to extend APM by limiting the number of edges in an MIP of $MIIA(t, \theta)$, to avoid incurring an unaccepted number of iterations in active friending.

3. RELATED WORK

Recommendation for passive friending has been explored in the past few years. Chen et al. [3] manifest that friending recommendations based on the topology of an on-line social network are the easiest way leading to the acceptance of an invitation. In contrast, recommendations based on contents posted by users are very powerful for discovering potential new friends with similar interests [3]. Meanwhile, research shows that preference extracted from social networking applications can be exploited for recommendations [15]. To avoid recommending socially distant candidates, users are allowed to specify different social constraints [23], e.g., the distance between a user and the recommended friending targets, to limit the scope of friending recommendation. Moreover, community information has been explored for recommendation [25]. Notice that the aforementioned research work and ideas are proposed for passive friending, where the friending targets are determined by the recommendation engines of social networking service providers in accordance with various criteria (e.g., preference and social closeness, etc). Thus, the user can conveniently (but *passively*) send an invitation to targets on the recommendation list. Complementary to the conventional passive friending paradigm, in this paper, we propose the notion of active friending where a friending target can be specified by the initiator. Accordingly, the recommendation service may assist and guide the initiator to *actively* approach a target.

The impact of social influence has been demonstrated in various applications, such as viral marketing [6, 17, 18] and interest inference [29]. Given an on-line social network, a major research problem is the seed selection problem, where the seeds correspond to the leaf nodes of MIA (i.e., initiator s and her friends) in our problem. In contrast, APM is to select the topology between the friends and t , instead of selecting the seeds. The homophily factor, capturing the tendency of users to connect with similar ones, has been considered in several applications, such that identifying trusted users [26] and users relationships [31] in social networks.

Notice that some works develop algorithms to return a subgraph or path, such as community detection [19], shortest path [10], pattern matching [11], or graph isomorphism query [8]. In contrast to the shortest path query, our algorithms for the APM problem make emphasize on returning a graph, instead of a path. The topology of the returned graph contains valuable neighborhood information of some common friends who can be leveraged to effectively increase the acceptance probability of a friending invitation. The initiator of a pattern matching or a graph isomorphism query needs to specify a subgraph as the query input. In contrast, this paper aims at finding an unknown graph between s and t to maximize the acceptance probability of a invitation to a friending target.

4. ALGORITHM DESIGN

To tackle the APM problem, we aim to design efficient algorithms in support of the invitation recommendations for

¹⁰MIA was proposed to simplify IC model, which is computation intensive and not scalable. Nevertheless, we prove that APM in IC model is NP-hard in Theorem 1 and not submodular by displaying a counter example in Appendix.

active friending. From our earlier discussions, it is easy to observe that the set of intermediate nodes in R , i.e., those to be recommended for invitation, play a crucial role in maximizing the acceptance probability for active friending. Here we first introduce a range-based greedy algorithm which provides some good insights for our other algorithms.

The algorithm, given an invitation budget r_R , aims to find the set of invitation candidates for recommendations to an initiator s who would like to make friends with a target t . Let R denote the answer set, which is initialized as empty at the beginning. The algorithm iteratively selects a node v from the neighbors of s 's current friends and adds it to R based on two heuristics: 1) the highest acceptance probability and 2) the number of remaining invitations. The former aims to minimize the potential waste of a friending invitation, while the latter avoids selecting a node too far away to reach t by constraining that v can only be at most $r_R - |R| - 1$ hops away from t . As a result, the range-based greedy algorithm is inclined to first expand the friend territory of s and then approach towards the neighborhood of t aggressively.

4.1 Selective Invitation with Tree Aggregation

While the range-based greedy algorithm is intuitive, the nodes added to R at separate iterations are not selected in a coordinated fashion. Thus, it is difficult for the range-based greedy algorithm to effectively maximize the acceptance probability. To address this issue, we propose a dynamic programming algorithm, call *Selective Invitation with Tree Aggregation (SITA)*, that finds the optimal solution for APM by exploring the maximum influence in-arborescence tree rooted at t (i.e., $MIIA(t, \theta)$) in a bottom-up fashion. SITA starts from the leaf nodes, i.e., nodes without in-neighbors, to explore $MIIA(t, \theta)$ in a topological order until t is reached finally. In order to obtain the optimal solution, SITA needs to explore various allocations of the r_R invitations to different nodes close to s or t in $MIIA(t, \theta)$. However, it is not necessary for SITA to enumerate all possible invitation allocations. Thanks to the tree structure of $MIIA(t, \theta)$, for each node v , SITA systematically summarizes the best allocation for v , i.e. which generates the highest acceptance probability for v , corresponding to the subtree rooted at v . The summaries will be exploited later by v 's parent node, i.e., the only out-neighbor of v , to identify the allocation generating the highest probability. The above procedure is repeated iteratively until t is processed, and the allocation of r_R invitations to the subtree rooted at t is the solution returned by SITA.

More specifically, let $f_{v,r}$ denote the maximum acceptance probability for v to accept the invitation from s while r invitations have been sent to the subtree rooted at v in $MIIA(t, \theta)$. By first sorting all nodes in topological order to t , we process $f_{v,r}$ of a node v after all $f_{u,r}$ of its in-neighbors u have been processed. Apparently, $f_{v,0} = 0$ for every node v that is not a friend of s because no invitation will be sent to the subtree rooted at v . In contrast, for every leaf node v , which is a friend of s (or s itself), $f_{v,r} = 1$ for $r = 0$. For all other nodes v in $MIIA(t, \theta)$, SITA derives $f_{v,r}$ according to each in-neighbor f_{u_i, r_i} as follows,

$$f_{v,r} = \max_{\sum r_i = r-1} \{1 - \prod_{u_i \in N^{in}(v)} [1 - f_{u_i, r_i} \cdot w_{u_i, v}]\}, \quad (1)$$

where $N^{in}(v)$ denotes the set of in-neighbors of v with

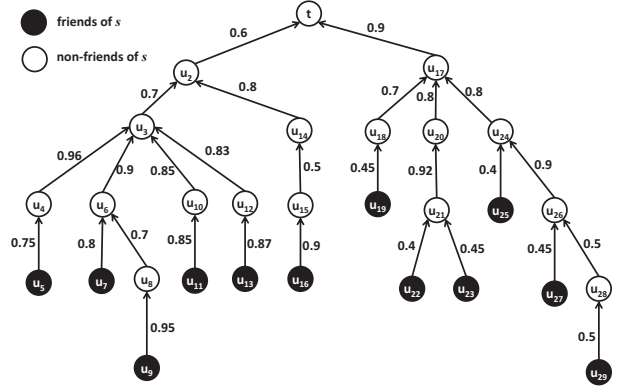


Figure 3: The running example(not including s and her edges)

$|N^{in}(v)| = d_v$, u_i is an in-neighbor of v , and r_i is the number of invitations sent from s to the subtree rooted at u_i . An invitation is sent to v , while the remaining $r - 1$ invitations are distributed to the in-neighbors of v . SITA effectively avoids examining all possible distribution of the $r - 1$ invitations to the nodes in the subtree. Instead, Eq. (1) examines only f_{u_i, r_i} of each in-neighbor u_i of v on every possible number of invitations r_i . In other words, only the in-neighbors of v , instead of all nodes in the subtree, participate in the computation of $f_{v,r}$ to efficiently reduce the computation involved. For each node v , $f_{v,r}$ is derived in ascending order of r until reaching $r = \min(r_R, z_v)$, where z_v is the number of nodes that are not friends of s in the subtree rooted at v . SITA stops after f_{t, r_R} is obtained. In the following, we show that SITA finds the optimal solution to APM.¹¹

LEMMA 1. *Algorithm SITA answers the optimal solution to APM.*

PROOF. We prove the lemma by contradiction. Assume that the solution from SITA, i.e., f_{t, r_R} , is not optimal. According to the recurrence, there must exist at least one in-neighbor $t_1 \in N_t^{in}$ together with the number of invitations r_1 such that f_{t_1, r_1} is not optimal. Similarly, since f_{t_1, r_1} is not optimal, there exists at least one in-neighbor $t_2 \in N_{t_1}^{in}$ of t_1 with the number of invitations r_2 such that f_{t_2, r_2} is non-optimal, $r_2 < r_1$. Here in the proof, let f_{t_i, r_i} denote the non-optimal solution found in i -th iteration of the above backtracking process, which will continue and eventually end with a probability f_{t_i, r_i} such that 1) t_i is a friend of s but $f_{t_i, 0} \neq 1$ or $f_{t_i, r_i} = 0$ for $r_i > 0$, or 2) t_i is not a friend of s but $f_{t_i, 0} \neq 0$. The above two cases contradict the initial assignment of SITA. The lemma follows. \square

Example. Figure 3 illustrates an example of $MIIA(t, \theta)$ with $r_R = 7$, where the nodes denote the users involved in deriving the maximal acceptance probability for t and the numbers labeled on edges denote the influence probability between two nodes. Without loss of generality, s and her homophily edges are not shown in Figure 3. Note that the dark nodes at the leaf are s and her existing friends and thus have the acceptance probability as 1, while the white nodes are the recommendation candidates to be returned by SITA

¹¹Due to the space constraint, we do not show the pseudocode of SITA here but refer the readers to the next section where a more general SITINA is presented.

along with their acceptance probabilities. SITA explores $MIIA(t, \theta)$ from the dark leaf nodes in a topological order, i.e., the $f_{v,r}$ of a node v is derived after all $f_{u,r}$ of its in-neighbors u are processed. Take u_4 as an example. $f_{u_4,0} = 0$ since no invitation is sent and $f_{u_4,1} = 1 - (1 - f_{u_5,0} \cdot 0.75) = 0.75$. Similarly, for u_8 , $f_{u_8,0} = 0$ and $f_{u_8,1} = 0.95$. Consider u_6 which has in-neighbors u_7 and u_8 , $f_{u_6,0} = 0$, $f_{u_6,1} = 0.8$, and $f_{u_6,2} = 1 - (1 - f_{u_7,0} \cdot 0.8)(1 - f_{u_8,1} \cdot 0.7) = 0.933$. Notice that for a node v , $f_{v,r}$ is derived for $r \in [0, \min(z_v, r_R)]$, e.g., for u_6 , we only derive $r \in [0, 2]$. Nevertheless, to find $f_{v,r}$, SITA needs to try different allocations by distributing the number of invitations r_i to each different neighbor u_i and then combining the solutions f_{u_i,r_i} to acquire $f_{v,r}$. For example, to derive $f_{u_{17},5}$, it is necessary to distribute 4 invitation to its in-neighbors, including u_{18} , u_{20} and u_{24} . The possible allocations for (r_{18}, r_{20}, r_{24}) include $(0, 1, 3)$, $(0, 2, 2)$, $(1, 0, 3)$, $(1, 1, 2)$ and $(1, 2, 1)^{12}$, which will obtain acceptance probability 0.5738, 0.7539, 0.7081, 0.6674 and 0.7639 respectively. Eventually, we obtain $f_{u_{17},5} = 0.7639$. Notice that the number of possible allocations grows exponentially. After all the nodes are processed, we obtain $f_{t,7} = 0.7483$. On the other hand, the greedy algorithm RG selects a user $v \notin R$ with the highest acceptance probability and at most $r_R - |R| - 1$ hops away from t . Accordingly, it selects u_8 , u_6 , u_{15} , u_{12} , and u_3 sequentially. In the 6th step, the node with the highest probability is u_{10} . However, u_{10} is 3-hops away with $3 > r_R - |R| - 1 = 2$ and thus not selected. Instead, it selects the node with the next highest acceptance probability, i.e., u_2 . In the last step, only the root t can be selected, so RG obtains a solution with the acceptance probability as 0.4013. As shown, SITA outperforms RG.

4.2 Selective Invitation with Tree and In-Node Aggregation

Unfortunately, SITA is not a polynomial-time algorithm because in Eq. (1), $O(r^{d_v})$ allocations are examined to distribute $r_i - 1$ invitations to the subtrees of the d_v in-neighbors corresponding to each node v . To remedy this scalability issue, we propose *Selective Invitation with Tree and In-Node Aggregation (SITINA)* to answer APM in polynomial time. SITINA effectively avoids processing of $O(r^{d_v})$ allocations by iteratively finding the best allocation for the first k in-neighbors, which in turn is then exploited to identify the best allocation for the first $k + 1$ in-neighbors. The process iterates from $k = 1$ till $k = d_v$. Consequently, the possible allocations for distributing $r_i - 1$ invitations to all in-neighbors are returned by Eq. (1) in $O(d_v r_R)$ time, where d_v is the in-degree of v in $MIIA(t, \theta)$.

To efficiently derive $f_{v,r}$ in Eq. (1), we number the in-neighbors of v as $u_1, u_2 \dots$ to u_{d_v} , where d_v is the in-degree of v . Let $m_{v,k,x}$ denote the maximum acceptance probability by sending x invitations to the subtrees of the first k neighbors of v , i.e., u_1 to u_k . Initially, $m_{v,1,x} = f_{u_1,x}$, $x \in [0, r_R]$. SITINA derives $m_{v,k,x}$ according to the best result of the first $k - 1$ in-neighbors,

$$m_{v,k,x} = \max_{x' \in [0, \min(z_{u_k}, x)]} \{1 - [1 - m_{v,k-1,x-x'}][1 - f_{u_k,x'} w_{u_k,v}]\}, \quad (2)$$

where $f_{u_k,x'} w_{u_k,v}$ is the acceptance probability for allocating x' invitations to the k -th in-neighbor u_k , and $m_{v,k-1,x-x'}$

¹²Some allocations are eliminated since $r_i \notin [0, \min(r_R, z_{u_i})]$.

k	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$
1	0.315	*	*	*	*	*
2	0.315	0.4931	0.6528	*	*	*
3	0.32	0.5342	0.6674	0.7639	0.8314	0.8520

Table 1: All $m_{u_7,k,x}$

is the best solution for allocating $x - x'$ invitations to the first $k - 1$ in-neighbors. By carefully examining different x' , we can obtain the best solution $m_{v,k,x}$ for a given k .

SITINA starts from $k = 1$ to $k = d_v$. For each k , SITINA begins with $x = 0$ until $k = \min(\sum_{i \in [1,k]} z_{u_i}, r_R - 1)$, where $\sum_{i \in [1,k]} z_{u_i}$ is the total number of nodes that are not friends of s in the subtrees of the first k in-neighbors. SITINA stops after finding every $m_{v,d_v,x}$, $x \in [0, \min(z_v, r_R - 1)]$. The pseudocode is presented in Algorithm 1, and the following lemma indicates that the optimal solution of APM is m_{t,d_t,r_R-1} .

LEMMA 2. For any v and r , $f_{v,r} = m_{v,d_v,r-1}$.

PROOF. We prove the lemma by contradiction. Assume that $m_{v,d_v,r-1}$ is not optimal. According to the recurrence, there exists at least one r_{d_v} such that $m_{v,d_v-1,(r-1-r_{d_v})}$ is not optimal. Similarly, since $m_{v,d_v-1,(r-1-r_{d_v})}$ is not optimal, there exists at least one r_{d_v-1} such that $m_{v,d_v-2,(r-1-r_{d_v}-r_{d_v-1})}$ is not optimal. Therefore, let $m_{v,d_v-i,(r-1-\sigma_i)}$, where $\sigma_i = \sum_{j \in [0,i-1]} r_{d_v-j}$, denote the non-optimal solution obtained in the i -th iteration. The backtracking process continues and eventually ends with $i = d_v - 1$, where $m_{v,1,r_1} \neq f_{u_1,x}$. It contradicts the initial assignment of $m_{v,1,r_1}$, and the lemma follows. \square

The following theorem proves that the algorithm answers the optimal solution to APM in $O(n_V r_R^2)$ time, where n_V is the number of nodes in a social network, and r_R is the number of invitations in APM. Note that any algorithm for APM is $\Omega(n_V)$ time because reading $MIIA(t, \theta)$ as the input graph requires $\Omega(n_V)$ time. Therefore, SITINA is very efficient, especially in a large social network with n_V significantly larger than d_{max} and r_R .

THEOREM 2. SITINA Algorithm answers the optimal solution to APM in $O(n_V r_R^2)$ time.

PROOF. According to Lemma 1 and Lemma 2, SITINA obtains the optimal solution of APM. Recall that n_V is the number of nodes in the social network, and d_v is the in-degree of a node v in $MIIA(t, \theta)$. The algorithm contains $O(n_V)$ iterations. Each iteration examines a node v to find $m_{v,d_v,x}$ for every $x \in [0, \min(z_v - 1, r_R - 1)]$, where r_R is number of invitations sent by s in APM. There are $O(d_v r_R)$ cases to be considered to explore all $m_{v,d_v,x}$ for v in Eq. (2), and each case requires $O(r_R)$ time. Therefore, finding $m_{v,d_v,x}$ for a node v needs $O(d_v r_R^2)$ time, and for all nodes in $MIIA(t, \theta)$ is $O(\sum_v d_v r_R^2)$, where $\sum_v d_v = |E|$. As $MIIA(t, \theta)$ is a tree (i.e. $|E| = n_V - 1$), the overall time complexity is $O(n_V r_R^2)$. The theorem follows. \square

Example. In the following, we illustrate how SITINA derives $f_{u_{17},r}$, $r \in [0, z_{u_{17}}]$. At the beginning, the in-neighbors of u_{17} are ordered as u_{18} , u_{20} and u_{24} .¹³ Then, we find all

¹³To avoid confusing, we keep their ID in this example without renaming them as $u_{u_{17}}^1$, $u_{u_{17}}^2$, and $u_{u_{17}}^3$.

Algorithm 1 Selective Invitation with Tree and In-Node Aggregation (SITINA)

Require: The query issuer s ; the targeted user t ; the influence tree $MIA(t, \theta)$ rooted at t ; the number of requests r_R that s can send.

Ensure: A set R of selected users that s sends requests to, such that the acceptance probability is maximized.

```

1: Obtain a topological order  $\sigma$  which orders a node without in-neighbor first.
2: for  $v \in \sigma$  do
3:   //obtain all  $f_{v,r}$ ,  $r \in [0, \min(r_R, n_v)]$ 
4:   Order in-neighbors of  $v$  as  $u_v^1, u_v^2, \dots, u_v^{d_v}$ 
5:    $m_{v,0,r} \leftarrow 0$  for  $\forall r \in [0, \min(r_R - 1, n_r - 1)]$ 
6:   for  $k = 1$  to  $d_v$  do
7:     for  $r = 1$  to  $\min(r_R - 1, n_v - 1)$  do
8:        $x = r - 1$ 
9:        $m_{v,k,x} = 0$ 
10:      for  $x' = 0$  to  $r$  do
11:        if  $m_{v,k,x} < 1 - [1 - m_{v,k-1,x-x'}][1 - f_{u_v^k,x'} w_{u_v^k,v}]$  then
12:           $m_{v,k,x} \leftarrow 1 - [1 - m_{v,k-1,x-x'}][1 - f_{u_v^k,x'} w_{u_v^k,v}]$ 
13:           $\pi_{v,k,x} \leftarrow x'$ 
14:         $f_{v,0} \leftarrow 0$ 
15:         $f_{v,x+1} \leftarrow m_{v,k,x}$ ,  $\forall x \in [0, \min(r_R - 1, n_r - 1)]$ 
16: Backtrack  $\pi_{v,k,x}$  to obtain  $R$ 
17: return  $R$  with maximized  $f_{t,r_R}$ 

```

$m_{u_{17},1,x} = f_{u_{18},x-1} w_{u_{18},u_{17}}$, $x \in [0, \min(z_{u_{18}}, r_R - 1)]$ first, representing the maximum acceptance probability u_{17} obtained by only sending x invitations to the subtree rooted at the first in-neighbor, i.e., u_{18} . Then we derive $m_{u_{17},2,x}$ for $x \in [0, \min(z_{u_{18}} + z_{u_{20}}, r_R - 1)]$ to acquire the maximum acceptance probability of u_{17} by sending invitations to subtrees rooted at u_{18} and u_{20} . Notice that different x' , representing the invitations distributed to the k -th subtree, needs to be examined in order to find the optimal solution. For instance, while deriving $m_{u_{17},3,4}$, we compare $1 - (1 - m_{u_{17},2,1})(1 - f_{u_{24},3} \times w_{u_{24},u_{17}}) = 0.7081$, $1 - (1 - m_{u_{17},2,2})(1 - f_{u_{24},2} \times w_{u_{24},u_{17}}) = 0.7539$ and $1 - (1 - m_{u_{17},2,3})(1 - f_{u_{24},1} \times w_{u_{24},u_{17}}) = 0.7417$ and obtain $m_{u_{17},3,4} = 0.7539$. After deriving all $f_{u_{17},x+1} = m_{u_{17},d_v,x}$ for $x \in [0, 6]$ ($\min(r_R - 1, z_{u_{17}} - 1) = 6$), the computation of u_{17} finishes. Table 1 lists the detailed results, where * denotes the instances with r exceeding the number of people who are not the friends of s in the first k subtrees.¹⁴

5. PERFORMANCE EVALUATION

We implement active friending in Facebook and conduct a user study and a comprehensive set of experiments to validate our idea of active friending and to evaluate the performance of the proposed algorithms. In the following, we first detail the methodology of our evaluation and then present the results of our user study and experiments, respectively.

5.1 Methodology

We adopt a user study and experiments, two complementary approaches, for the performance evaluation. We aim to use the user study to investigate how the recommendation-based active friending approach is faring with the approach

based on the users' own strategies (i.e., which they would follow under the existing environment of social networking services). To perform the user study, we implement an app. on Facebook. Through the app., the user is able to decide whom to invite based on their own strategies to approach the target. Meanwhile, according to the recommendations generated from the Range-Based Greedy (RG) algorithm and the Selective Invitation with Tree and In-Node Aggregation (SITINA) algorithm, respectively, the user also sends alternative sets of invitations to proceed the active friending activities for comparison.¹⁵ Note that Selective Invitation with Tree Aggregation (SITA) is not considered because it makes exactly the same recommendations as SITINA. We recruited 169 volunteers to participate in the user study. Each volunteer is given 25 targets with varied invitation budgets to work on. The social distances between the volunteer and the targets are pre-determined in order to collect results for comparison under controlled parameter settings.

On the other hand, we conduct experiments by simulation to evaluate the solution quality and efficiency of SITA, SITINA, and RG, implemented in an HP DL580 server with four Intel Xeon E7-4870 2.4 GHz CPUs and 128 GB RAM. Two large real datasets, *FacebookData* and *FlickrData* are used in the experiments. *FacebookData* contains 60,290 users and 1,545,686 friend links crawled from Facebook [28], and *FlickrData* contains 1,846,198 users and 22,613,981 friend links crawled from Flickr [22]. The initiator s and target t are selected uniformly at random.

An important issue faced in both of our user study and the experiments is the social influence and homophily factors captured in the social network, which are required for RG, SITA and SITINA to make recommendations. Most of previous works adopt a fixed probability (e.g., 1/degree in [17, 6, 4, 7]) or randomly choose a probability from a set of values (e.g., 0.001, 0.01, 0.1 in [6, 4]) due to the lack of real social influence probabilities and homophily probabilities. To address this issue, in the user study, we obtain the social influence probability on each edge by mining the interaction history of volunteers in Facebook in accordance with [13, 14]. We also derive the homophily probabilities from s to other nodes by mining the profile information in their Facebook pages based on [3]. The social network in the user study is denoted as *UserStudyData*. As for the social networks in *FacebookData* and *FlickrData* that are to be used for experiments, we unfortunately do not have personal profiles and historical interactions of the nodes. Thus, we could not generate the social influence probability and homophily probability by mining real data. As a result, we choose to assign the link weights of the social network based on: i) the distributions of social influence and homophily probabilities obtained from our user study (denoted as US), and ii) the Zipf distribution for its ability to capture many phenomena studied in the physical and social sciences [32].

5.2 User Study

Through the user study, we have logged the responses of participants to invitations and thus are able to calculate the acceptance probabilities corresponding to invitations under various circumstances. Using the collected data, we make a number of comparisons.

¹⁴Note that $m_{u_{17},k,x} = 0$ when $x = 0$.

¹⁵To alleviate the burden of the participants, we send invitations on their behalves to the recommended candidates.

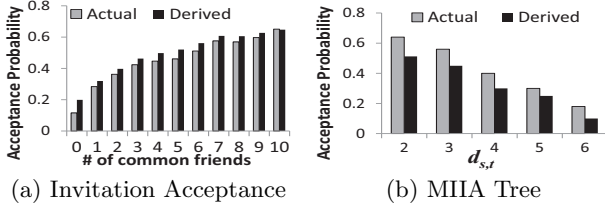


Figure 4: Verify the derived acceptance probability

First, we would like to verify that the acceptance probability of an active friending plan derived based on MIIA tree (using the mined social influence and homophily probabilities as the link weights) are consistent with that of the plan being executed in the user study. Towards this goal, we first verify the accuracy of our invitation acceptance model (for single invitation) by comparing the derived acceptance probability and the actual acceptance probability obtained from real activities in the user study. Figure 4(a), where results obtained from the user study and our model are respectively labeled as *Actual* and *Derived*, plots the comparison in terms of the number of common friends in an invitation. As shown, the acceptance probabilities of both User and Model increase as the number of common friends in invitations increases. Most importantly, the results are consistently close, showing our invitation acceptance model (and the social influence and homophily weights used) are able to reasonably capture the decision making upon invitations in real life.

Notice that the above comparison focusses on the aspect of invitation acceptance only, without taking into account the social network topology, which we approximate with the MIIA tree. To verify that using the MIIA tree is sufficiently effective for active friending planning, we further compare the acceptance probability derived using our proposed algorithms and the actual acceptance probability obtained through executing the plan in the user study. Figure 4(b) shows that, under various distance between initiator and target, the acceptance probabilities derived using MIIA tree is reasonably close to the actual acceptance probabilities.

Next, we compare the effectiveness of strategies based on RG, SITINA and the participants' own heuristics. Figure 5(a) plots the comparison by varying the number of friending invitations, r_R . RG and SITINA generally outperform user heuristics (labeled as User) under all settings. We can observe that the performance of SITINA is generally very good and getting better as r_R increases, while the performance of User and RG have a leap from $r_R = 5$ to 10 and remain close afterwards. This indicates the extra computation effort required for deriving recommendations due to the increased invitation budget are worthwhile, outperforming the heuristic strategies derived based on RG and human intuition. Figure 5(b) evaluates the acceptance probability of t under varied settings of $d_{s,t}$. When $d_{s,t}$ is 2, it is more likely to have a lot of common friends (due to the nature of social networks) and thus getting better acceptance probabilities. When $d_{s,t}$ increases, it becomes more difficult for an initiator to make effective decisions due to the less number of common friends and the lack of knowledge about the larger and more complex social network topology behind. As shown in Figure 5, SITINA has the best performance.

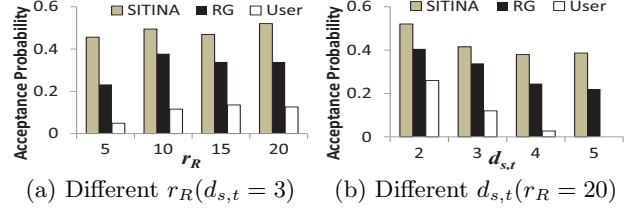


Figure 5: Acceptance probability in user study

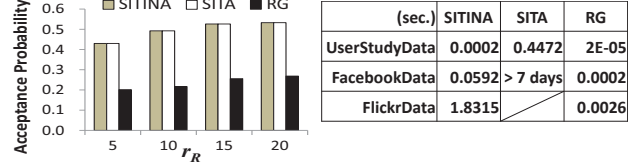


Figure 6: Acceptance probability ($d_{s,t} = 2$) Figure 7: Average Run-time ($r_R = 20$)

5.3 Experimental Results

While the user study verifies that SITINA is able to achieve the best performance, the size of social network is small due to the limited number of volunteers participating in the study. To further validate our ideas in a large-scale social network and to evaluate the scalability of SITINA, we conduct an experimental study by simulations.

5.3.1 Scalability

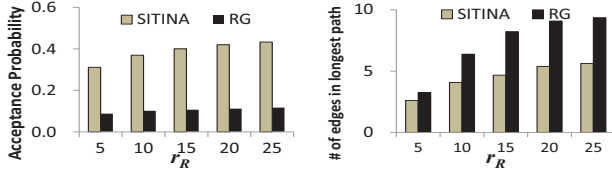
As proved earlier, SITA can obtain the optimal solution of APM. However, it is not scalable as it needs to examine all combinations of invitation allocations. Here we use it as a baseline to compare the efficacy and efficiency with SITINA over social networks of different sizes. First, we compare the results by randomly sampling 50 (initiator, target) pairs using *UserStudyData*. As Figure 6 depicts, both SITA and SITINA significantly outperform RG in terms of acceptance probability. Next, we compare their running time, not only using *UserStudyData* but also the large-scale *FacebookData* and *FlickrData*. As shown in Figure 7, the SITA algorithm takes more than 7 days without returning the answer and thus not feasible for practical use. For the rest of experiments, we only compare SITINA with RG.

5.3.2 Sensitivity Tests

In this section, we conduct a series of sensitivity tests to examine the impact of different parameters, including the invitation budget (r_R), the number of friends of s (N), the distance between the initiator and target ($d_{s,t}$), and the skewness of social influence and homophily probabilities (α). In experiments on the impacts of r_R , N , $d_{s,t}$, we have tested both the FacebookData (US) and FlickrData (US).¹⁶ As the observations on both datasets are quite similar, we only report both results for the first experiment and skip the FlickrData result for the rest due to space constraint. Finally, in the last experiment, we use FacebookData (ZF) to observe how α may potentially impact our algorithms.

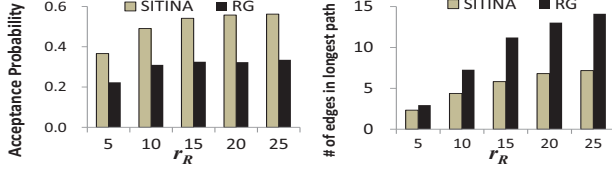
Impact of r_R . By varying r_R and setting the default $d_{s,t}$

¹⁶US and ZF denote the link weights assigned based on models from User Study and Zipfian Distribution.



(a) Acceptance probability (b) Number of edges

Figure 8: Varying r_R (*FacebookData* (US))



(a) Acceptance probability (b) Number of edges

Figure 9: Varying r_R (*FlickrData* (US))

of sampled (s, t) pairs as 4, we compare SITINA and RG in terms of the acceptance probability and the number of iterations using *FacebookData* (US) and *FlickrData* (US) (see Figure 8 and Figure 9, respectively). As shown in Figure 8(a) and 9(a), SITINA exhibits much better performance than RG, regardless of the r_R . Meanwhile, Figure 8(b) and 9(b) manifest that the longest path in the solution R obtained by SITINA is shorter than that in RG because RG tends to spend invitations on some local users with higher acceptance probabilities.¹⁷

Impact of N . We are interested in finding whether the number of friends of s has an impact on the performance. Thus, we choose four different groups of initiators s (who have around 100, 200, 300, and 400 friends, respectively) and sample 100 different targets t to compare their acceptance probabilities. With r_R set as 25, Figure 10 shows that as the number of friends increases, the initiators have more choices to reach their targets. As shown, SITINA can find the optimal solution with high acceptance probability, while near-sighted RG tends to select the friends of friends with higher acceptance probabilities and eventually results in small acceptance probability to t .

Impact of $d_{s,t}$. We also conduct an experiment to understand the impact of $d_{s,t}$ on the performance. Not surprisingly, the finding is consistent with our user study (please refer to Section 5.2 and Figure 5(b)). Thus, we do not plot the result here due to the space constraint.

Impact of α . In the experiments above, social influences and homophily factors are modeled based on our user study, but the distributions in different social networks may vary. Thus, through the skewness parameter α , we use Zipf distribution, to examine the impact of α on our algorithms. As shown in Figure 11, we can observe that as the distributions of social influence and homophily become more skewed (i.e., α increases), the acceptance probabilities of SITINA and RG drops, because it becomes more difficult for invitations to get accepted when there are less number of highly

¹⁷RG is inclined to take more time to reach t because invitations are sequentially sent towards t . The latency of friending a new intermediate node is different for each node.

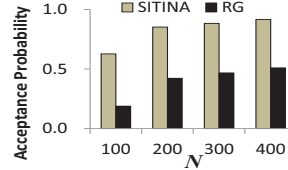


Figure 10: Varying N (*FacebookData* (US))

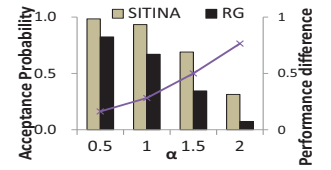


Figure 11: Varying α (*FacebookData* (ZF))

influential links while the number of less influential links increases. It is also worth noting that, as the line in the figure indicates, the percentage of performance difference between SITINA and RG (i.e., acceptance prob. of SITINA divided by that of RG) increases, showing that SITINA is able to handle skewed distribution much better than RG.

6. CONCLUSION AND FUTURE WORK

Observing the need of active friending in everyday life, this paper formulates a new optimization problem, named Acceptance Probability Maximization (APM), for making friending recommendations on-line social networks. We propose Algorithm Selective Invitation with Tree and In-Node Aggregation (SITINA), to find the optimal solution for APM and implement SITINA in Facebook. User study and experimental results manifest that active friending can effectively maximize the acceptance probability of the friending target.

In our future work, we will first explore the impact of delay between sending an invitation and acquiring the result in active friending. This is important when the user would like to make friends with the target within certain time frame. In addition, for multiple friending targets, it is not efficient to configure recommendations separately for each target. An idea is to give priority to the intermediate nodes that can approach many targets simultaneously. We will study active friending of a group of targets in the future work.

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APPENDIX

We display that APM is not submodular by the following counter example with four users. The user a is the existing friend of s , i.e., $S = \{a\}$, while b , c , and t are non-friend users of s . The influence probability is labeled beside each edge. Consider adding a new user c to two different set of selected users $R_S = \{t\}$ and $R_T = \{b, t\}$, where $R_S \subset R_T$. If the submodular property holds, $ap(t, S, R_S \cup \{c\}, MIIA(t, \theta)) - ap(t, S, R_S, MIIA(t, \theta)) \geq ap(t, S, R_T \cup \{c\}, MIIA(t, \theta)) - ap(t, S, R_T, MIIA(t, \theta))$ should hold. The acceptance probability of selecting R_S , i.e., $ap(t, S, R_S, MIIA(t, \theta))$, is 0 since there is no path from a to t . Similarly, $ap(t, S, R_S \cup \{c\}, MIIA(t, \theta)) = 0$. The acceptance probability of selecting R_T is $1 - (1 - 0.9 \times 0.1) = 0.09$, and adding c into R_T results in acceptance probability $ap(t, S, R_T \cup \{c\}, MIIA(t, \theta)) = 1 - (1 - 0.09)(1 - 0.9) = 0.909$. However, $ap(t, S, R_S \cup \{c\}, MIIA(t, \theta)) - ap(t, S, R_S, MIIA(t, \theta)) = 0 < ap(t, S, R_T \cup \{c\}, MIIA(t, \theta)) - ap(t, S, R_T, MIIA(t, \theta)) = 0.909 - 0.09 = 0.819$. There is a counter example and the submodular property does not hold in APM.

